Earning While Learning

How to Run Batched Bandit Experiments

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Based on joint work with Jan Kemper and Gaul et al. (2024)

Agenda

Adaptive experimental designs and algorithms

- ε-first
- ε-greedy
- Thompson sampling

Adaptive experiments in practice

- Data structure
- Inference about causal effects

The bbandits command

- Syntax, options, returned results
- Empirical applications
- Monte Carlo simulation

Conclusions

Adaptive experimental designs

- Randomized controlled trials gold standard of causal inference
- Adaptive experiments allow “earning while learning”
- Push to replace non-adaptive randomized trials with bandits
  - In medicine, economics, political science, survey methods research, education, psychology, ...
  - Practitioners use bandit algorithms
  - Can improve outcomes for participants (optimize regret)
  - Can improve policies learned at the end of trial (best-arm identification)
- Some popular algorithms
  - ε-first
  - ε-greedy
  - Thompson sampling

Recent “exploding” growth or papers

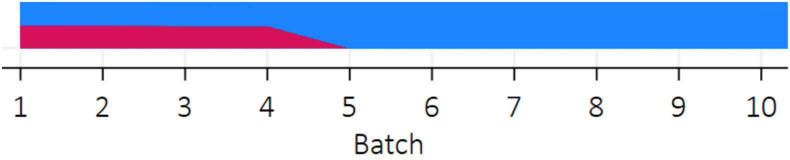
- In medicine (Lei et al., 2022)
- economics and finance (Hirano and Porter, 2023; Chen and Andrews, 2023; Kasy and Sautmann, 2021; Hadad et al., 2021; Avivi et al., 2021)
- political science (Offer-Westort et al., 2021)
- survey methods research (Gaul et al., 2024)
- education (Rafferty et al., 2019)
- psychology (Schulz et al., 2020)
- ...
- Practitioners use bandit algorithms
  (Hill et al., 2017; Scott, 2015; Agarwal et al., 2014; Chapelle and Li, 2011; Scott, 2010; Graepel et al., 2010)

Stylized data structure

Obs	Selected arm	Reward
1	A	0
2	B	0
3	A	1
4	B	0
5	A	0
6	B	1
7	A	1
8	B	0
9	A	0
10	A	1
11	A	1
12	B	0
13	A	1
14	A	0
15	A	1
16	B	0

- Does arm A or arm B perform better?
- Which arm to play in next trial (round 17)?

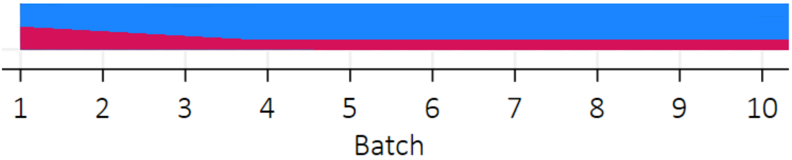
ε-first



- Epsilon-first is widely known as A/B testing
- Often applied to two-armed bandits
- The first ε share of trials serve as exploration or burn-in phase
- Researchers assign a uniform share of participants to each arm
- Estimate each arm's outcome predicts future outcomes
- In the remaining $(1 - \varepsilon)$ share of trials (exploitation phase)

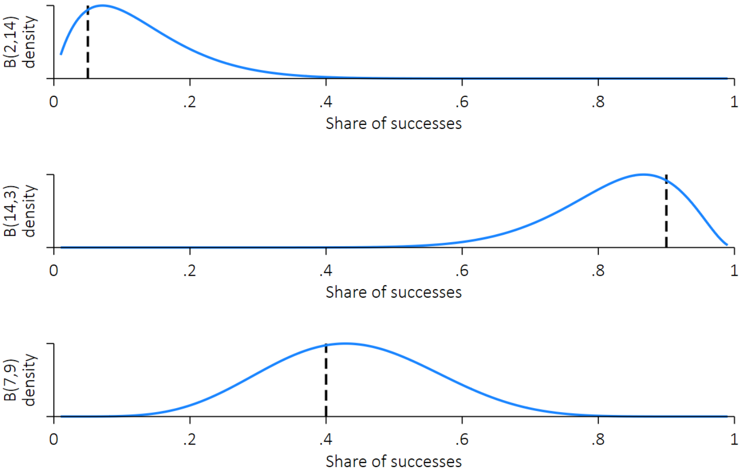
  $\rightarrow$ only the arm with the best empirical estimate is selected

ε-greedy



- Explores for the entire number of trials of the experiment
- Selects best treatment for share $(1-\varepsilon)$ of all trials
- Share can be constant or decreasing
- Assigns all treatment arms with equal probability for share ε
- Even after having learned about the average outcome of each arm, constant epsilon-greedy explores some epsilon fraction of the trials $\rightarrow$ asymptotically no convergence to optimal arm

Thompson (1933, 1935) sampling



- Beta-Bernoulli Thompson sampling
- Models uncertainty about the shape of the distribution and the expected outcome *R* explicitly

Click to watch!

Thompson sampling

- Distribution (not only expectation) is updated according to Bayes' rule
- Probability at trial t that a given arm k provides optimal reward is

$$P(E_\theta[R_k, t] = \max\{E_\theta[R_1], \ldots, E_\theta[R_K]\}|R_t) =$$

$$\int_0^1 \mathbb{1}\left[S_t = \arg\max_{k=1,\ldots,K} E_\theta[R_k, t]\right] P(\theta|R_t) d\theta,$$

  where S_t is single arm at trial t for which exp. reward is maximized
- Update prior distribution to get posterior distribution $P(\theta|R_t)$

Thompson sampling

- Beta distribution $B(R_{k,t}|\alpha_k, \beta_k)$ denotes the density of the beta distribution for random variable R_t with parameters α_k and β_k
- Posterior distribution $P(\theta|R_t)$ is also beta with parameters that can be updated according to a simple rule:

$$(\alpha_k, \beta_k) = \begin{cases} (\alpha_k, \beta_k) & \text{if chosen arm} \neq k, \\ (\alpha_k, \beta_k) + (R_t, 1 - R_t) & \text{if chosen arm} = k. \end{cases}$$

- α_k or β_k increases by one with each observed success or failure
- Distribution more concentrated as $\alpha_k + \beta_k$ grows
- Mean $\alpha_k/(\alpha_k + \beta_k)$ and variance $\frac{\alpha_k \beta_k}{(\alpha_k+\beta_k)^2(\alpha_k+\beta_k+1)}$

Bandits >> A/B Tests

- Push to replace non-adaptive randomized trials with bandits
  - In development and labor economics, finance, biostats, health, ...
  - Can improve outcomes for participants (optimize regret)
  - Can improve policies learned at the end of trial (best-arm identification)
- **Problem**:
  - Bandits are not easy to implement

    Not available in statistical software like Stata
  - Bandits break inference

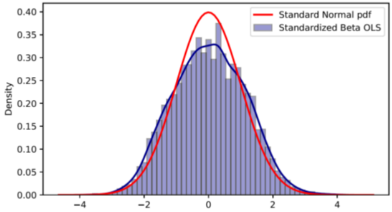
    Adaptive arm allocations

    $\rightarrow$ breaks asymptotics of usual estimators

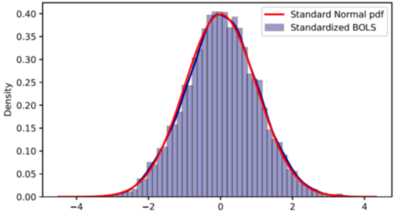
    $\rightarrow$ wrong confidence intervals
- **Solution: Batched OLS (BOLS) for Batched Bandits**

A simple example

- OLS and BOLS under Beta-Bernoulli two-arm Thompson Sampling with batch size $N_t = 100$ at batch $t = 10$
- All simulations are with no margin ($\beta_1 = \beta_0 = 0$)



(a) Empirical distribution of standardized OLS estimator for the margin



(b) Empirical distribution of standardized BOLS estimator for the margin

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	.
2	B	0	.
3	A	0	.
4	B	0	.
5	.	1	.
6	.	1	.
7	.	1	.
8	.	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	.	1	.
6	.	1	.
7	.	1	.
8	.	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	.
6	B	1	.
7	A	1	.
8	B	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	.
10	A	2	.
11	A	2	.
12	B	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	A	3	.
14	A	3	.
15	A	3	.
16	B	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	A	3	1
14	A	3	0
15	A	3	1
16	B	3	0

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward
1	A	0	0	0.5
2	B	0	0	0.2
3	A	0	1	0.5
4	B	0	0	0.2
5	A	1	0	0.5
6	B	1	1	0.2
7	A	1	1	0.5
8	B	1	0	0.2
9	A	2	0	0.5
10	A	2	1	0.2
11	A	2	1	0.5
12	B	2	0	0.2
13	A	3	1	0.5
14	A	3	0	0.2
15	A	3	1	0.5
16	B	3	0	0.2

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS
1	A	0	0	0.5	0.600
2	B	0	0	0.2	0.167
3	A	0	1	0.5	0.600
4	B	0	0	0.2	0.167
5	A	1	0	0.5	0.600
6	B	1	1	0.2	0.167
7	A	1	1	0.5	0.600
8	B	1	0	0.2	0.167
9	A	2	0	0.5	0.600
10	A	2	1	0.2	0.600
11	A	2	1	0.5	0.600
12	B	2	0	0.2	0.167
13	A	3	1	0.5	0.600
14	A	3	0	0.2	0.600
15	A	3	1	0.5	0.600
16	B	3	0	0.2	0.167

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS	Batch-Wise OLS
1	A	0	0	0.5	0.600	0.500
2	B	0	0	0.2	0.167	0.000
3	A	0	1	0.5	0.600	0.500
4	B	0	0	0.2	0.167	0.000
5	A	1	0	0.5	0.600	0.500
6	B	1	1	0.2	0.167	0.500
7	A	1	1	0.5	0.600	0.500
8	B	1	0	0.2	0.167	0.500
9	A	2	0	0.5	0.600	0.667
10	A	2	1	0.2	0.600	0.667
11	A	2	1	0.5	0.600	0.667
12	B	2	0	0.2	0.167	0.000
13	A	3	1	0.5	0.600	0.667
14	A	3	0	0.2	0.600	0.667
15	A	3	1	0.5	0.600	0.667
16	B	3	0	0.2	0.167	0.000

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS	Batch-Wise OLS	ω_t
1	A	0	0	0.5	0.600	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
2	B	0	0	0.2	0.167	0.000	$\sqrt{\frac{2\times 2}{2+2}}$
3	A	0	1	0.5	0.600	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
4	B	0	0	0.2	0.167	0.000	$\sqrt{\frac{2\times 2}{2+2}}$
5	A	1	0	0.5	0.600	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
6	B	1	1	0.2	0.167	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
7	A	1	1	0.5	0.600	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
8	B	1	0	0.2	0.167	0.500	$\sqrt{\frac{2\times 2}{2+2}}$
9	A	2	0	0.5	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
10	A	2	1	0.2	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
11	A	2	1	0.5	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
12	B	2	0	0.2	0.167	0.000	$\sqrt{\frac{1\times 3}{1+3}}$
13	A	3	1	0.5	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
14	A	3	0	0.2	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
15	A	3	1	0.5	0.600	0.667	$\sqrt{\frac{1\times 3}{1+3}}$
16	B	3	0	0.2	0.167	0.000	$\sqrt{\frac{1\times 3}{1+3}}$

Point estimates OLS vs. BOLS

Aggregate or batched OLS (BOLS) estimator

$$\Delta^{\text{BOLS}} = \frac{\sum_t^T \omega_t \times \Delta_t^{BOLS}}{\sum_t^T \omega_t},$$

where $\omega_t = \sqrt{\frac{N_{t,k} \times N_{t,b}}{N_{t,k} + N_{t,b}}}$.

- weights batchwise estimates
- such that the aggregate margins are consistent and asymptotically **normally** distributed (Zhang et al., 2020)

Point estimates OLS vs. BOLS

Example from stylized data structure

OLS	$\widehat{\text{Reward}} = 0.6 - 0.433 \times \mathbb{1}_{\text{arm B}}$
BOLS	$-0.443 = \frac{1\times 0.5 + 1\times 0 + \sqrt{\frac{1\times 3}{1+3}}\times 0.667 + \sqrt{\frac{1\times 3}{1+3}}\times 0.667}{1+1+\sqrt{\frac{1\times 3}{1+3}}+\sqrt{\frac{1\times 3}{1+3}}}$
	$\widehat{\text{Reward}} = 0.6 - 0.443 \times \mathbb{1}_{\text{arm B}}$

Inference OLS vs. BOLS

$$Pr\left(\Delta^{\text{BOLS}} - c\sigma w_t \leq \mu \leq \Delta^{\text{BOLS}} + c\sigma w_t\right) = 1 - \alpha,$$

- where Δ^{BOLS} is the weighted estimated marginal effect
- μ is the hypothesized difference between means of the arms
- c is a critical value, e.g., the $1 - \alpha/2 = 97.5$th percentile of a normal
- σ reflects the sampling error
- w_t is a weight correcting the bias due to adaptive sampling

$$w_t = \sqrt{T} / \sum_{t=1}^{T} \omega_t.$$

- T is the total number of batches
- $N_{t,k}$ is the number of times that comparison arm k was played
- $N_{t,b}$ is the number of times that baseline arm b was played

The bbandits command

Syntax & Options

Click to download!

- bbandits *reward assignedarm batch, options*

Returned results

- OLS margins
- BOLS margins
- *z* statistics
- p-values
- BOLS 95% confidence intervals
- observations of the reference arm
- observations of the treatment arm
- treatment arm indicator and the OLS 95% confidence intervals

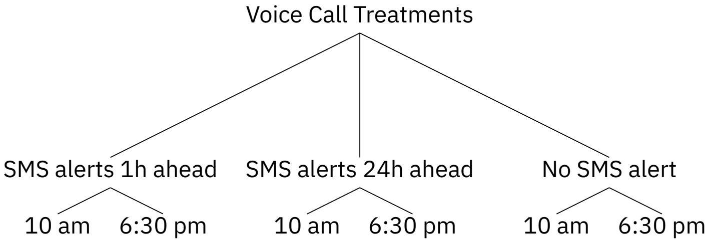
Empirical application (Kasy and Sautmann, 2021)

Six call methods to enroll rice farmers

- Kasy and Sautmann (2021) designed an experiment using exploration sampling for Precision Agriculture for Development
- NGO that works with government partners to provide a phone-based personalized agricultural extension service to farmers in India
- Aim is to choose best call methods to enroll rice farmers in one state

Six call methods to enroll rice farmers

- The outcome (reward) is a binary variable for call completion:
  - = 1 if call recipient answered five questions asked during call
  - = 0 otherwise



Empirical application (Kasy and Sautmann, 2021)

- Exploration sampling replaces the Thompson assignment shares
- modification shifts weight away from the best performing option to competing treatments
- 10,000 valid phone numbers randomly assigned to one of 16 batches
- batch size was 600 numbers each (and one with 400)
- From June 3, 2019 batches run every other day, completed next day

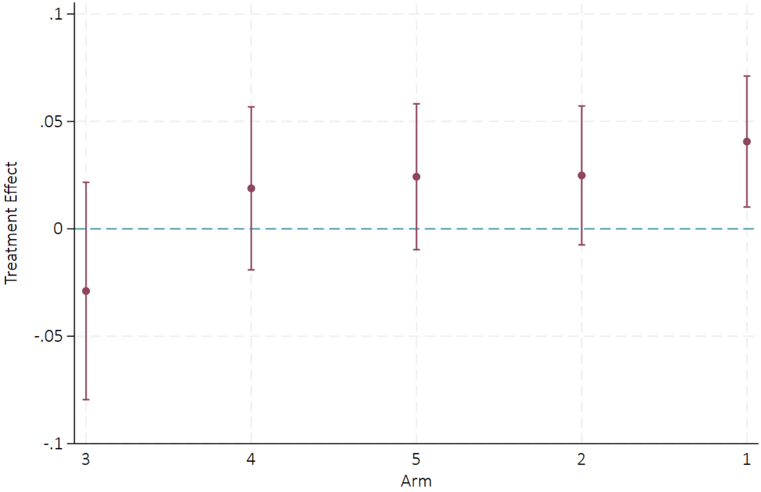
Empirical application (Kasy and Sautmann, 2021)

```
.
. use "example data\kasy_sautmann_2021.dta", clear
. bbandits outcome treatment date
```

Number of obs	=	10000				
Est. Rewards only best arm	=	1926	Mean reward best arm	=	0.1926
Actual total reward	=	1804	Actual mean reward	=	0.1804
Est. reward uniformly chosen arms	=	1709	Mean reward uniform	=	0.1709

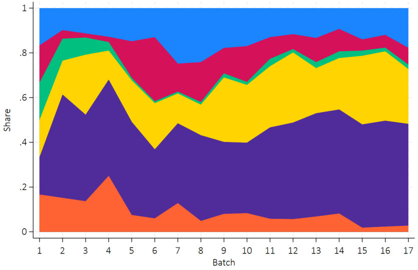
Arm b	Mean Reward						Share arm b
	0.1606						0.0903
k v. b	**Margin OLS**	**Margin BOLS**	**z**	**P>|z|**	**[95% Conf.**	**Interval]**	**Share arm k**
1-0	0.0320	0.0406	2.61	0.009	0.0101	0.0711	0.3931
2-0	0.0185	0.0249	1.51	0.132	-0.0075	0.0572	0.2234
3-0	-0.0158	-0.0289	-1.12	0.262	-0.0795	0.0216	0.0366
4-0	0.0078	0.0188	0.97	0.330	-0.0191	0.0568	0.1081
5-0	0.0192	0.0243	1.40	0.161	-0.0097	0.0582	0.1485

Empirical application (Kasy and Sautmann, 2021)

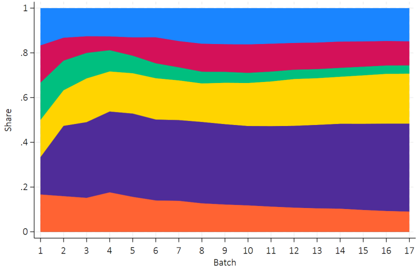


The figure was generated using `kasy_sautmann_2021.dta` and running `bbandits outcome treatment date`

Empirical application (Kasy and Sautmann, 2021)



(a) Batchwise shares



(b) Cumulative shares

The figure was generated using `kasy_sautmann_2021.dta` and running `bbandits outcome treatment date`

Empirical application (Kasy and Sautmann, 2021)

Takeaways

Clear best and worst arms

- Best: Calling farmers at 10 am after a message an hour ahead of time
- Worst: Calling at 6:30 pm without a text message alert

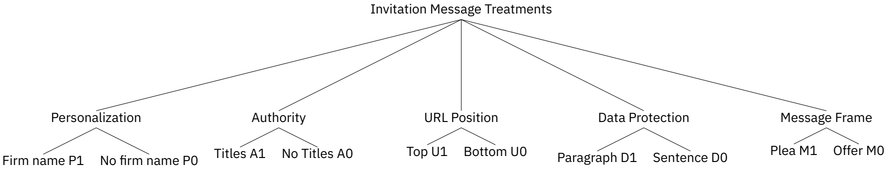
Improvement of success rate

- 18.04% success rates within the experiment
- 17.15% success rate with equal assignment

32 invitation messages for business survey

- Gaul et al. (2024) designed an experiment using Thompson sampling to support the German Business Panel (GBP)
- Aim is to select among a variety of different invitation messages to survey firm decision makers in Germany
- The GBP is a web-based survey study of firm decision makers in Germany that invites participants each work day (see Bischof et al. (2024); Hack and Rostam-Afschar (2024))
- The outcome (reward) is a binary variable for the start of the survey:
  - = 1 if email invitation recipient started the survey
  - = 0 otherwise

Empirical application (Gaul et al., 2024)



- Five components of invitation letters and their full interactions
  $\rightarrow \quad 2^5 = 32$ treatments
  - **personalization** by mentioning or not mentioning the firm name
  - **authority** of the sender by listing the official full academic titles along with the senders' names or their names only
  - **URL position** to start the survey at the top or bottom of the invitation
  - **data protection** in a separate paragraph with two strongly phrased sentences or in a single sentence
  - **message frame** by including phrases that plea for support in the survey's cause or to simply offer to participate

Empirical application (Gaul et al., 2024)

- 11,000 randomly selected contacts from firms in Germany
- Assigned to each of 15 batches from a list of 176,000 contacts
- Each batch corresponds to a week between
  August 16, 2022 and November 25, 2022
- First four batches used fixed and balanced burn-in phase
  with treatment probability 1/32
- From batch 5, Thompson assignment rule for each consecutive batch

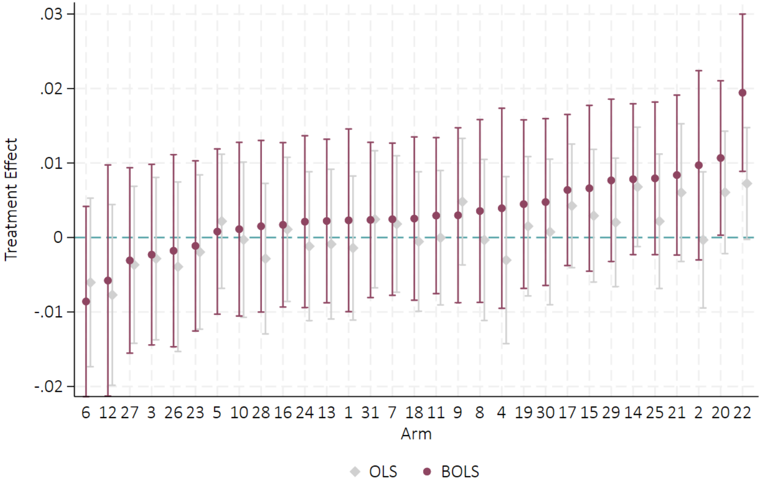
Empirical application (Gaul et al., 2024)

```
. use "example data\gaul_et_al_2024.dta", clear
. bbandits reward selected trial
```

Number of obs	=	176000			
Est. Rewards only best arm	=	8623	Mean reward best arm	=	0.0490
Actual total reward	=	7833	Actual mean reward	=	0.0445
Est. reward uniformly chosen arms	=	7430	Mean reward uniform	=	0.0422

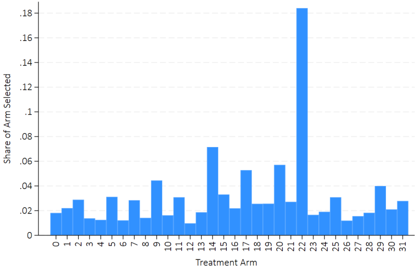
Arm b	Mean Reward						Share arm b
	0.0417						0.0181
k v. b	**Margin OLS**	**Margin BOLS**	**z**	**P>|z|**	**[95% Conf.**	**Interval]**	**Share arm k**
1-0	-0.0014	0.0023	0.37	0.712	-0.0100	0.0146	0.0220
2-0	-0.0003	0.0097	1.50	0.135	-0.0030	0.0224	0.0288
3-0	-0.0028	-0.0023	-0.37	0.710	-0.0144	0.0098	0.0137
4-0	-0.0030	0.0039	0.57	0.567	-0.0095	0.0174	0.0125
5-0	0.0022	0.0008	0.14	0.888	-0.0103	0.0119	0.0312
6-0	-0.0060	-0.0086	-1.32	0.187	-0.0214	0.0042	0.0121
7-0	0.0018	0.0025	0.47	0.638	-0.0078	0.0127	0.0284
8-0	-0.0003	0.0036	0.57	0.570	-0.0087	0.0158	0.0141
9-0	0.0048	0.0030	0.50	0.619	-0.0088	0.0147	0.0444
10-0	-0.0003	0.0011	0.19	0.851	-0.0105	0.0128	0.0162
11-0	-0.0000	0.0029	0.55	0.583	-0.0075	0.0134	0.0308
12-0	-0.0077	-0.0058	-0.73	0.466	-0.0213	0.0097	0.0097
13-0	-0.0009	0.0022	0.40	0.692	-0.0088	0.0132	0.0186
14-0	0.0068	0.0078	1.51	0.130	-0.0023	0.0180	0.0715
15-0	0.0029	0.0066	1.16	0.245	-0.0045	0.0177	0.0331
16-0	0.0011	0.0017	0.30	0.762	-0.0093	0.0127	0.0219
17-0	0.0042	0.0064	1.23	0.218	-0.0038	0.0165	0.0527
18-0	-0.0005	0.0025	0.45	0.650	-0.0084	0.0135	0.0255
19-0	0.0015	0.0045	0.78	0.438	-0.0068	0.0158	0.0256
20-0	0.0061	0.0107	2.02	0.044	0.0003	0.0210	0.0571
21-0	0.0060	0.0084	1.53	0.126	-0.0024	0.0191	0.0271
22-0	0.0072	0.0194	3.61	0.000	0.0089	0.0300	0.1840
23-0	-0.0020	-0.0011	-0.19	0.847	-0.0126	0.0103	0.0166
24-0	-0.0012	0.0021	0.36	0.718	-0.0094	0.0137	0.0190
25-0	0.0022	0.0079	1.52	0.129	-0.0023	0.0182	0.0308
26-0	-0.0039	-0.0018	-0.27	0.787	-0.0147	0.0111	0.0119
27-0	-0.0037	-0.0031	-0.48	0.628	-0.0155	0.0094	0.0155
28-0	-0.0028	0.0015	0.26	0.797	-0.0100	0.0130	0.0183
29-0	0.0020	0.0077	1.38	0.168	-0.0032	0.0186	0.0400
30-0	0.0007	0.0048	0.83	0.405	-0.0064	0.0160	0.0210
31-0	0.0024	0.0024	0.44	0.658	-0.0081	0.0128	0.0278

Empirical application (Gaul et al., 2024)

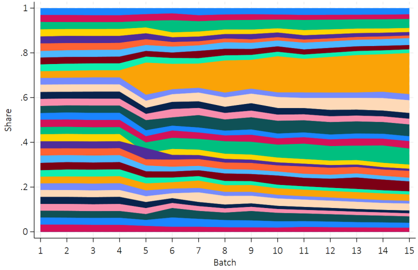


The figure was generated using `gaul_et_al_2024.dta` and running `bbandits reward selected trial`

Empirical application (Gaul et al., 2024)



(a) Total frequency of treatment assignment



(b) Cumulative shares of arms played

The figure was generated using `gaul_et_al_2024.dta` and running
`bbandits reward selected trial`

Empirical application (Gaul et al., 2024)

Takeaways

Clear best and worst arms

- Using personalization, no authority, top URL, no emphasis on data protection, and pleading for support has greatest success (arm 22)
- Success rate 6.11% with BOLS, 4.89% with OLS
- 18.40% of firm decision makers received the best invitation
- Only 1.21% received least successful invitation
- Compared to not personalizing, pers. increases starting rate by 9.95%

Interaction effects important, too

- High authority increases starting rate by 0.16 (p-value: 0.039)
- Pleading for help increases by 0.25 percentage p. (p-value: 0.004)

Monte Carlo Simulations

- Bernoulli-Thompson or Epsilon-Greedy
- Vary clipping rate
- Number of observations per batch N_t
- True difference between the two arms $\Delta[R]$
- Average of the 10000 calculated differences between the two arms
- 95%-type-I error rates
- Under normality H_0 should be rejected in only 5%

Monte Carlo Simulations

- *Click to watch*: OLS fails normality when margin is small
- *Click to watch*: BOLS normal even when margin is small
- Run own simulations with

```
bbandit_sim 0.5 0.4 0.3, size(200) batch(10) clipping(0.1)
Thompson plot_Thompson
```

Best Practices

- Report OLS and BOLS
- BOLS inference in the small margin case **correct** but...
- OLS inference in the large margin case **more precise**
- Check batch-wise OLS estimates
- **At least 50 observations** per batch and arm
- From *statistical testing* perspective:

  more observations per batch and arm better
- from *regret optimization* perspective

  fewer observations and thus fails are better
- use **bbandits** to simulate, visualize, and analyse bandit experiments

Conclusions

- Bandits may improve learning and exploitation
- There is a push to use more bandits in real experiments in development and labor econ, biostats, health, ...
- need for valid inference to support conclusions
  - bandits break inference
  - researchers wand valid confidence intervals
- **Batched bandit inference (BBandit)**
  - First Stata routine for adaptive experiments
  - allows valid statistical inference & correct coverage for batched bandits
  - easy illustrations for statistical learning from adaptively collected data

Thank you!
https://rostam-afschar.de/

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