RLearning:
Short guides to reinforcement learning

Unit 1-5: Inference with Batched Bandits

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How to get bandits normal?

Stylized data structure

Obs	Selected arm	Reward
1	A	0
2	B	0
3	A	1
4	B	0
5	A	0
6	B	1
7	A	1
8	B	0
9	A	0
10	A	1
11	A	1
12	B	0
13	A	1
14	A	0
15	A	1
16	B	0

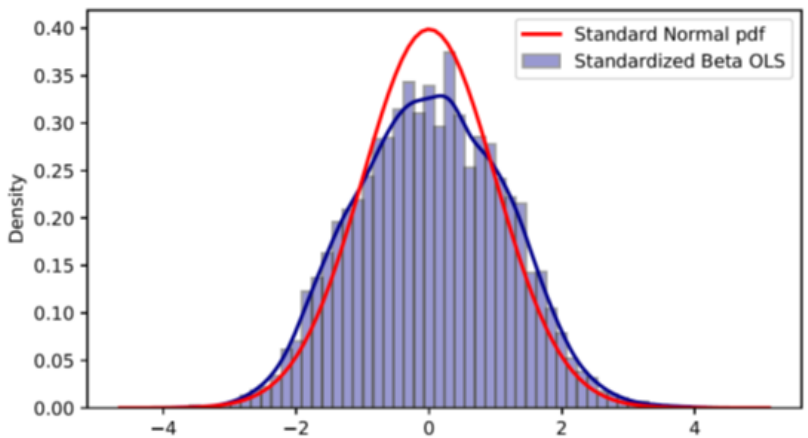
- Does arm A or arm B perform better?
- Which arm to play in next trial (round 17)?

Bandits >> A/B Tests

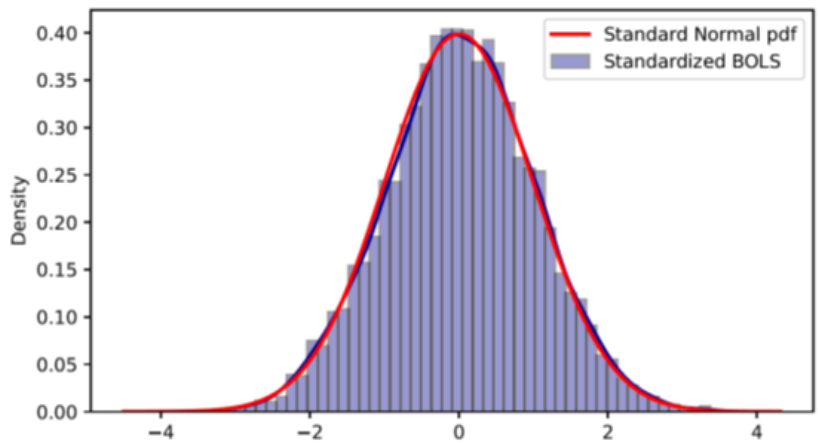
- Push to replace non-adaptive randomized trials with bandits
  - In development and labor economics, finance, biostats, health, ...
  - Can improve outcomes for participants (optimize regret)
  - Can improve policies learned at the end of trial (best-arm identification)
- **Problem:**
  - Bandits are not easy to implement
    Not available in statistical software like Stata
  - Bandits break inference
    Adaptive arm allocations
    $\rightarrow$ breaks asymptotics of usual estimators
    $\rightarrow$ wrong confidence intervals
- **Solution: Batched OLS (BOLS) for Batched Bandits**

A Simple Example

- OLS and BOLS under Beta-Bernoulli two-arm Thompson Sampling with batch size $N_t = 100$ at batch $t = 10$
- All simulations are with no margin ($\beta_1 = \beta_0 = 0$)



(a) Empirical distribution of standardized OLS estimator for the margin



(b) Empirical distribution of standardized BOLS estimator for the margin

Batchwise Data Collection

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	.
2	B	0	.
3	A	0	.
4	B	0	.
5	.	1	.
6	.	1	.
7	.	1	.
8	.	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	.	1	.
6	.	1	.
7	.	1	.
8	.	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	.
6	B	1	.
7	A	1	.
8	B	1	.
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	.	2	.
10	.	2	.
11	.	2	.
12	.	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	.
10	A	2	.
11	A	2	.
12	B	2	.
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	.	3	.
14	.	3	.
15	.	3	.
16	.	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	A	3	.
14	A	3	.
15	A	3	.
16	B	3	.

Stylized data structure

Obs	Selected arm	Batch	Reward
1	A	0	0
2	B	0	0
3	A	0	1
4	B	0	0
5	A	1	0
6	B	1	1
7	A	1	1
8	B	1	0
9	A	2	0
10	A	2	1
11	A	2	1
12	B	2	0
13	A	3	1
14	A	3	0
15	A	3	1
16	B	3	0

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward
1	A	0	0	0.5
2	B	0	0	0.2
3	A	0	1	0.5
4	B	0	0	0.2
5	A	1	0	0.5
6	B	1	1	0.2
7	A	1	1	0.5
8	B	1	0	0.2
9	A	2	0	0.5
10	A	2	1	0.2
11	A	2	1	0.5
12	B	2	0	0.2
13	A	3	1	0.5
14	A	3	0	0.2
15	A	3	1	0.5
16	B	3	0	0.2

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS
1	A	0	0	0.5	0.600
2	B	0	0	0.2	0.167
3	A	0	1	0.5	0.600
4	B	0	0	0.2	0.167
5	A	1	0	0.5	0.600
6	B	1	1	0.2	0.167
7	A	1	1	0.5	0.600
8	B	1	0	0.2	0.167
9	A	2	0	0.5	0.600
10	A	2	1	0.2	0.600
11	A	2	1	0.5	0.600
12	B	2	0	0.2	0.167
13	A	3	1	0.5	0.600
14	A	3	0	0.2	0.600
15	A	3	1	0.5	0.600
16	B	3	0	0.2	0.167

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS	Batch-Wise OLS
1	A	0	0	0.5	0.600	0.500
2	B	0	0	0.2	0.167	0.000
3	A	0	1	0.5	0.600	0.500
4	B	0	0	0.2	0.167	0.000
5	A	1	0	0.5	0.600	0.500
6	B	1	1	0.2	0.167	0.500
7	A	1	1	0.5	0.600	0.500
8	B	1	0	0.2	0.167	0.500
9	A	2	0	0.5	0.600	0.667
10	A	2	1	0.2	0.600	0.667
11	A	2	1	0.5	0.600	0.667
12	B	2	0	0.2	0.167	0.000
13	A	3	1	0.5	0.600	0.667
14	A	3	0	0.2	0.600	0.667
15	A	3	1	0.5	0.600	0.667
16	B	3	0	0.2	0.167	0.000

Stylized data structure

Obs	Selected arm	Batch	Reward	True Expected Reward	OLS	Batch-Wise OLS	ω_t
1	A	0	0	0.5	0.600	0.500	$\sqrt{\frac{2\times2}{2+2}}$
2	B	0	0	0.2	0.167	0.000	$\sqrt{\frac{2\times2}{2+2}}$
3	A	0	1	0.5	0.600	0.500	$\sqrt{\frac{2\times2}{2+2}}$
4	B	0	0	0.2	0.167	0.000	$\sqrt{\frac{2\times2}{2+2}}$
5	A	1	0	0.5	0.600	0.500	$\sqrt{\frac{2\times2}{2+2}}$
6	B	1	1	0.2	0.167	0.500	$\sqrt{\frac{2\times2}{2+2}}$
7	A	1	1	0.5	0.600	0.500	$\sqrt{\frac{2\times2}{2+2}}$
8	B	1	0	0.2	0.167	0.500	$\sqrt{\frac{2\times2}{2+2}}$
9	A	2	0	0.5	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
10	A	2	1	0.2	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
11	A	2	1	0.5	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
12	B	2	0	0.2	0.167	0.000	$\sqrt{\frac{1\times3}{1+3}}$
13	A	3	1	0.5	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
14	A	3	0	0.2	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
15	A	3	1	0.5	0.600	0.667	$\sqrt{\frac{1\times3}{1+3}}$
16	B	3	0	0.2	0.167	0.000	$\sqrt{\frac{1\times3}{1+3}}$

Point estimates OLS vs. BOLS

Aggregate or batched OLS (BOLS) estimator

$$\Delta^{\text{BOLS}} = \frac{\sum_t^T \omega_t \times \Delta_t^{BOLS}}{\sum_t^T \omega_t},$$

where $\omega_t = \sqrt{\frac{N_{t,k} \times N_{t,b}}{N_{t,k} + N_{t,b}}}$.

- $N_{t,k}$ is the number of times that comparison arm k was played
- $N_{t,b}$ is the number of times that baseline arm b was played
- weights batchwise estimates
- such that the aggregate margins are consistent and asymptotically **normally** distributed (**?**)

Point estimates OLS vs. BOLS

Example from stylized data structure

OLS	$\widehat{\text{Reward}} = 0.6 - 0.433 \times \mathbb{1}_{\text{arm B}}$
BOLS	$-0.443 = \frac{1 \times 0.5 + 1 \times 0 + \sqrt{\frac{1 \times 3}{1+3}} \times 0.667 + \sqrt{\frac{1 \times 3}{1+3}} \times 0.667}{1 + 1 + \sqrt{\frac{1 \times 3}{1+3}} + \sqrt{\frac{1 \times 3}{1+3}}}$
	$\widehat{\text{Reward}} = 0.6 - 0.443 \times \mathbb{1}_{\text{arm B}}$

Inference OLS vs. BOLS

$$\mathbb{P}\left(\Delta^{\text{BOLS}} - c\sigma w_t \leq \mu \leq \Delta^{\text{BOLS}} + c\sigma w_t\right) = 1 - \alpha,$$

- where Δ^{BOLS} is the weighted estimated marginal effect
- μ is the hypothesized difference between means of the arms
- c is a critical value, e.g., the $1 - \alpha/2 = 97.5$th percentile of a normal
- σ reflects the sampling error
- w_t is a weight correcting the bias due to adaptive sampling

$$w_t = \sqrt{T} / \sum_{t=1}^{T} \omega_t.$$

- T is the total number of batches

Batched Bandits in Practice

Empirical application (**?**)

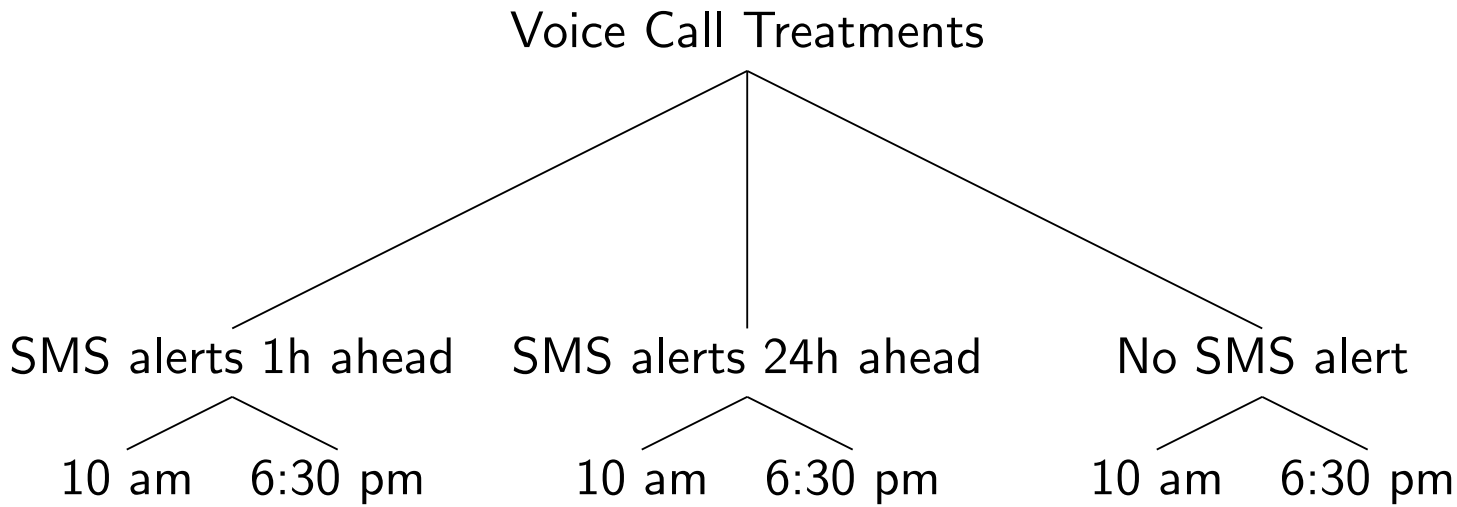
Six call methods to enroll rice farmers

- **?** designed an experiment using *exploration sampling* for Precision Agriculture for Development
- NGO that works with government partners to provide a phone-based personalized agricultural extension service to farmers in India
- Aim is to choose best call methods to enroll rice farmers in one state

Empirical application (**?**)

Six call methods to enroll rice farmers

- The outcome (reward) is a binary variable for call completion:
  - $= 1$ if call recipient answered five questions asked during call
  - $= 0$ otherwise



Empirical application (**?**)

- *Exploration sampling* replaces the Thompson assignment shares
- modification shifts weight away from the best performing option to competing treatments
- 10,000 valid phone numbers randomly assigned to one of 16 batches
- batch size was 600 numbers each (and one with 400)
- From June 3, 2019 batches run every other day, completed next day

Empirical application (**?**)

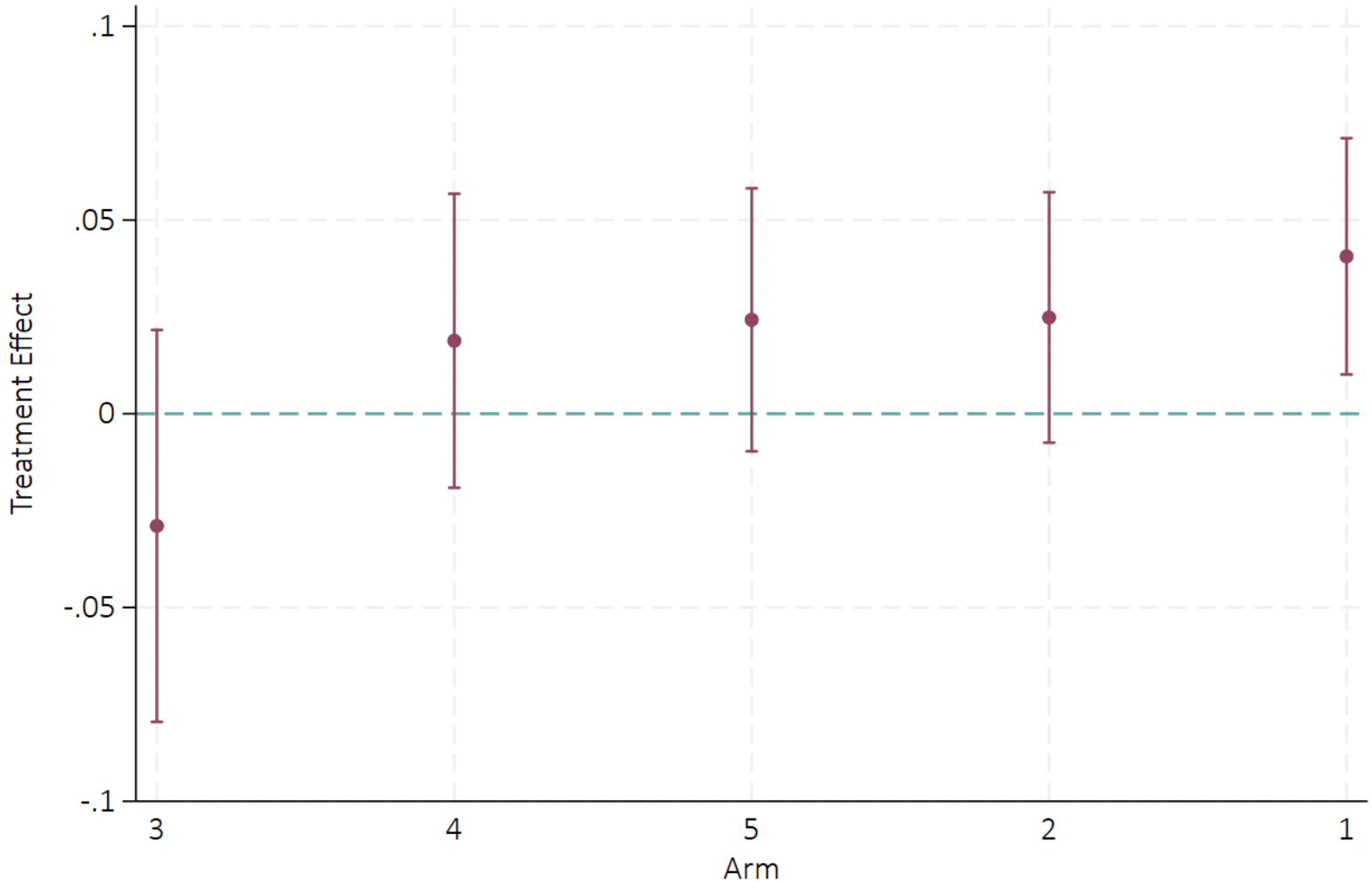
```
.
. use "example data\kasy_sautmann_2021.dta", clear
. bbandits outcome treatment date

Number of obs                      =        10000
Est. Rewards only best arm         =         1926    Mean reward best arm      =       0.1926
Actual total reward                =         1804    Actual mean reward        =       0.1804
Est. reward uniformly chosen arms  =         1709    Mean reward uniform       =       0.1709
```

Arm b	Mean Reward						Share arm b
	0.1606						0.0903
k v. b	**Margin OLS**	**Margin BOLS**	**z**	**P>|z|**	**[95% Conf.**	**Interval]**	**Share arm k**
1-0	0.0320	0.0406	2.61	0.009	0.0101	0.0711	0.3931
2-0	0.0185	0.0249	1.51	0.132	-0.0075	0.0572	0.2234
3-0	-0.0158	-0.0289	-1.12	0.262	-0.0795	0.0216	0.0366
4-0	0.0078	0.0188	0.97	0.330	-0.0191	0.0568	0.1081
5-0	0.0192	0.0243	1.40	0.161	-0.0097	0.0582	0.1485

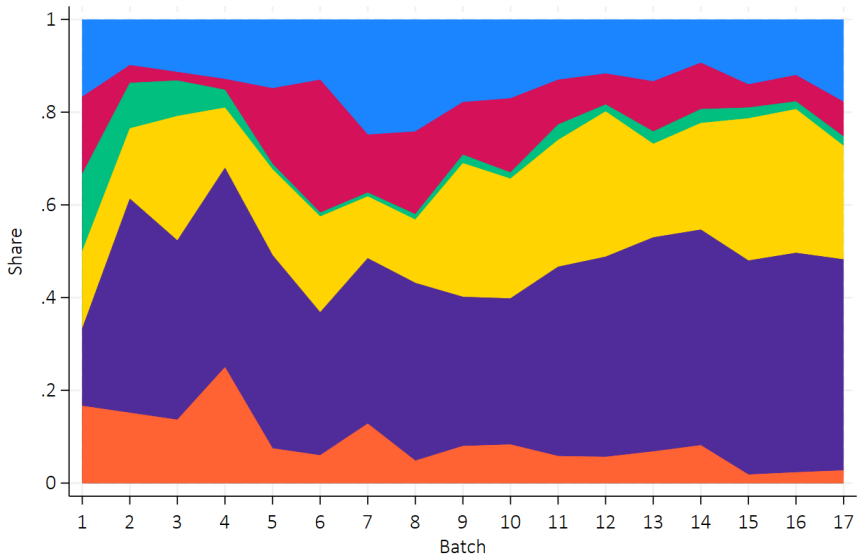
Treatment	0	1	2	3	4	5
SMS	—	1h ahead	24h ahead	—	1h ahead	24h ahead
Call time	10 am	10 am	10 am	6:30 pm	6:30 pm	6:30 pm

Empirical application (**?**)

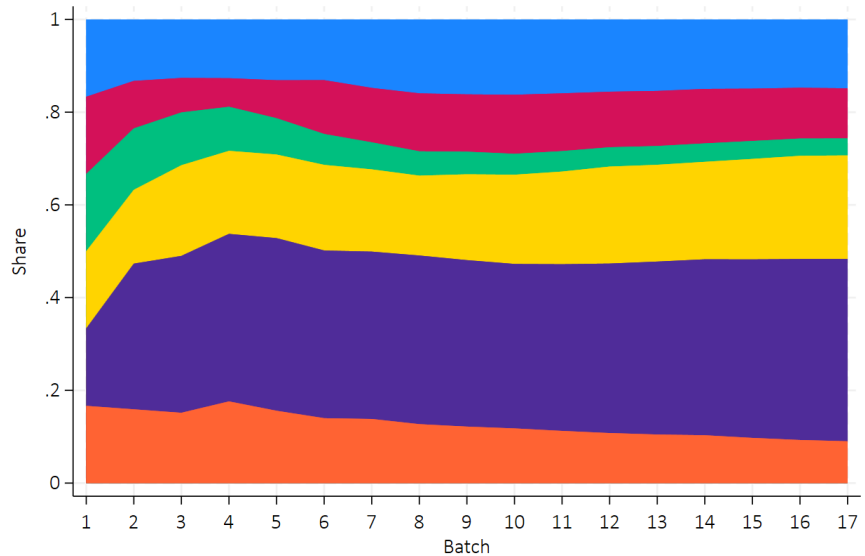


The figure was generated using `kasy_sautmann_2021.dta` and running `bbandits outcome treatment date`

Empirical application (?)



(a) Batchwise shares



(b) Cumulative shares

The figure was generated using `kasy_sautmann_2021.dta` and running

```
bbandits outcome treatment date
```

Empirical application (?)

Takeaways

Clear best and worst arms

- Best: Calling farmers at 10 am after a message an hour ahead
- Worst: Calling at 6:30 pm without a text message alert

Improvement of success rate

- 18.04% success rates within the experiment
- 17.15% success rate with equal assignment

References I

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Offer-Westort, M., A. Coppock, and D. P. Green (2021): "Adaptive Experimental Design: Prospects and Applications in Political Science," *American Journal of Political Science*, 65(4), 826–844.

Zhang, K., L. Janson, and S. Murphy (2020): "Inference for Batched Bandits," *Advances in neural information processing systems*, 33, 9818–9829.

Takeaways

How to run Batched Bandit Experiments?

- Bandits may improve learning and exploitation
- There is a push to use more bandits in real experiments in economics, biostats, health, psychology, political science (**?**), . . . and survey research methods!
- need for valid inference to support conclusions
  - bandits break inference
  - researchers want valid confidence intervals
- **Batched bandit inference (download BBandits)**
  - (**?**)
  - BOLS allows valid statistical inference & correct coverage for batched bandits